5. MORPHOLOGY, ENTROPY, AND STABILITY IN NETWORKED STRUCTURES

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1. INTRODUCTION

This paper concerns the exploration of the relation between the morphology (concentration and connectivity), the entropy, and the stability of networked structures. In section 2 we will introduce the network morphology concept. In section 3 we will address two approaches to network characterization: Traditional network measures and the concept of entropy to characterize relative order in a network. In section 4 we will link the entropy concept to the network characteristics. It will be shown that entropy will grow steeply if a certain balance between connectivity and concentration is disturbed. While sections 2 through 4 are concerned with a static networked structure, section 5 will consider the dynamics of networks. We explore the relation between the network characteristics and the ‘spontaneous’ instability of simple non-linear dynamic systems. Section 6 concludes with implications and directions for further research.

2. MORPHOLOGY OF ECONOMIC NETWORKS

Every network has a morphology. Morphology is defined as the form and structure of a network. The morphology of a network can be described by two separate elements: connectivity and concentration.

2.1 Connectivity and concentration

The connectivity of a network can be defined as the relationship between the number of nodes and the number of connections between the nodes. The higher the number of connections with respect to the number of nodes, the higher the connectivity.

Concentration defines the number of connections between a certain node and the others. The higher the number of connections from one node to all the others, the higher the con-centration. The measurement of concentration has a relationship with the kurtosis of the distribution of connections among the various nodes.

We have defined a network as a structure consisting of nodes and links. Concentration and connectivity provide information over the network, they have a certain relationship, as shown in figure 1. Networks with a high connectivity and a high concentration cannot exist. This would imply that every node is connected to every other node, but still nodes exist that have more connections than others. The same reasoning can be done for medium concentration / high connectivity and medium connectivity / high concentration networks. They also cannot exist. Obviously the border areas between high, medium and low are somewhat fuzzy.

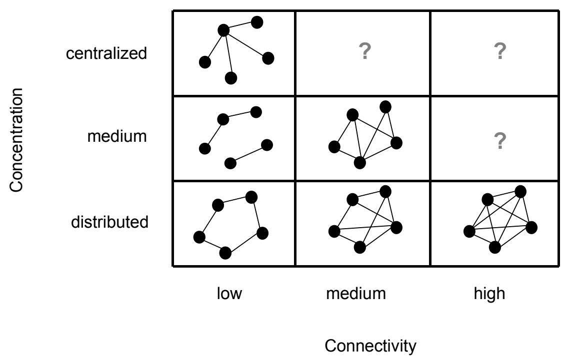

Fig. 1. Relation between network connectivity and concentration

Let us relate these abstract network measures to economic networks, such as business organizations. The morphology concept can be applied to social systems by analyzing the links between social entities. Different configurations yield different levels of order / disorder. In this way order in social systems can be seen as an expression of the existence of meaningful and purposeful relationships between functional elements of such a system. Without such relationships the whole of the system can have no meaning or purpose. In such case the whole is identical to the sum of parts and no synergy or common purpose can exist. The principles of order through fluctuations were first formulated in thermodynamics. The central idea is that self-organizing systems do not solely thrive on order, they need a certain amount of chaos. If the system fixes itself in a certain configuration, it will no longer be adaptive. It follows that a certain amount of disorder should be present for the system to remain adaptive. In other words, the system should have a certain level of entropy, somewhere between order and chaos. In an optimally adaptive system order and variety (chaos) are in an optimal balance. Neither can be reduced without reducing the system’s adaptability.

It is a popular belief that networked structures exists because of the ability or even necessity for all agents to relate to all other agents. Yet it can be shown that a high connectivity factor of a system (the average number of links any agent in the network has), combined with a low concentration factor (there are no concentration points) leads to a very rich ‘solution space’ but and increasing inability to find a suitable solution. In other words, if the number of degrees of freedom in relation to new solutions is larger than the complexity of the problem itself, the payback will rapidly decay as opposite to the conventional Taylorist situation. This, in turn, is an example of under-complexity, in which the ‘solution space’ of the organization is too small for the complexity of the outside world. Here, there is a low connectivity factor, combined with a high concentration factor.

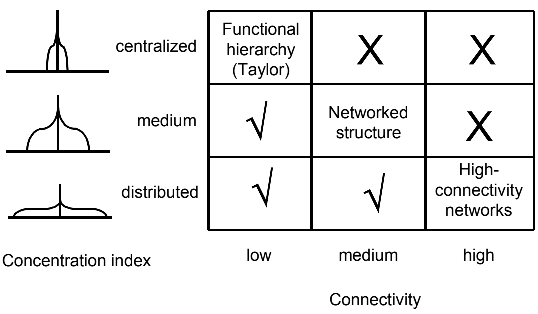

Fig. 2. Connectivity and concentration in economic networks

2.2 Example

Let us consider a social network structure of 10 entities, say employees in a business organization.

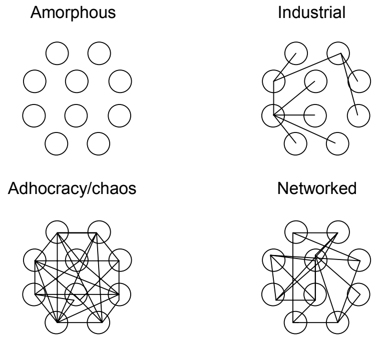

Fig. 3. Example of an economic network structure (employees in an organization)

In this figure, four possible characteristics of relations between these 10 entities are indicated. These links could be expressions of co-operation and/or communication between employees. For simplicity reasons we assume a digital situation: links exist or don't exist, they are bi-directional and they are uni-dimensional. All co-operation and communication below a certain threshold is supposed not to be existent, and consequently the links drawn in figure indicate strong co-operation and/or communication.

In the left-hand example, no links exist, and as no links exist between the entities there cannot be a common objective, or meaningful identity, associated with such organization. They are merely 10 individuals apparently arbitrarily isolated from the universe and put together on this paper. This way of arranging entities we would not call organization, but a complete absence of any form of organization.

In the second example in the figure one of the entities is connected to most other entities. Apparently this one entity is in the center of what the structure is intended for, and is apparently the beginning and the end of all activities undertaken by the structure. We will readily recognize the existence of hierarchy in this structure, as the central entity apparently is necessarily governing the behaviour of the other entities.

In the third example all entities are connected to all other entities. In this situation where apparently all entities interact with the same intensity with all other entities, there is no structure visible. Structure which would indicate a way in which these entities relate to each other in any peculiar way, and which could provide a clue with respect to the purpose, learning and working of the organization. In fact if all relations are equal, then apparently all entities are universal or completely identical and if this is the case, it is difficult to see why they would need to relate to each other, other than exploiting each other's capacity in response to some outside force.

In the fourth example the connectivity is substantially higher than in the hierarchical structure but substantially less than in the third example. Here a rich pattern of connections exists, suggesting some sort of meaning of relations between the various entities of the organization. And this meaning will most likely reflect the purpose of the organizational structure as a whole, as well as the differences in identity and capabilities of the individual entities.

3. APPROACHES TO NETWORK CHARACTERIZATION

Different approaches may be taken to characterize networks. We will discuss the two that are helpful to the reasoning in the next paragraphs: traditional network analysis and the entropy measure. This can be applied to organizational structures by analyzing the structure of the distribution of links between organizational entities. Different configurations yield different levels of concentration and connectivity, and different levels of organizational entropy (an expression of order / disorder).

3.1 Traditional network analysis

Network analysis offers a means for bridging the gap between macro- and micro-level explanations of social structures. Research design for network analysis consists of four elements [1]:

- The choice of sampling units, i.e. the actual network and the nodes that will be studied. The delimitation of network boundaries depends to a great extent upon a researcher's purposes. In our case, the sampling units consist of economic networks, e.g., business organizations, supply chains, or markets.
- The form of relations, referring to a) the intensity or strength of the relation between two agents, and b) the level of joint involvement in the same activities. For simplicity we assume relation to be digital: they either exist, or they don't, there are no 'levels'.
- The relational content, e.g. transaction relations, communication relations, sentiment relations, authority / power relations. Here also for simplicity we assume relations to be one-dimensional.
- The level of data analysis. Four conceptually distinct levels of analysis can be distinguished: 1) the egocentric network, or the relations of a single agent within the network (generating n units of analysis at sample size n); 2) the level of dyadic relationships, i.e. formed by a pair of nodes (generating $(n^2-n)/2$ units of analysis at sample size n); 3) the level of triad relationships, i.e. formed by three nodes and their linkages (generating n/3 distinct triads at sample size n); 4) the complete network, using complete information of relations among all agents. In this paper, we study economic networks at the fourth level, searching for the characteristics of the network as a whole.

We are fully aware that the result of our choices in the elements mentioned above constitutes a very basic approach to network characterization. Hereby we largely ignore a broad spectrum of network theory in sociology and economics. Our approach connects however to the more 'mathematical' literature that tries to apply quantitative measures of network structures.

The standard traditional measures are the ones we started with in the previous paragraph: connectivity and concentration, where connectivity is:

$$Connectivity = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}}{\left(n^2-n\right)/2} \qquad i \neq j$$

Concentration is a bit more complicated [1]. For even a simple measure, it is first necessary to have the relative centrality per agent or node in the network. For calculating this relative centrality we need g_{ij}, which is the number of geodesics linking *i* and *j*, and g_{imj}, which is the number of geodesics linking *i* and *j* that involve point m:

$$Centrality\ (p_m) = \frac{2\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{g_{imj}}{g_{ij}}}{n^2-3n+2} \qquad i \neq j$$

Subsequently we calculate the sum of the difference between the centrality of the most central actor C(p*) and the centrality of all other actors $C(p_i)$:

$$Centralisation = \frac{\sum_{i=1}^{n}\left(C(p*)-C(p_i)\right)}{n^3-4n^2+5n-2}$$

3.2 Organizational entropy

Measuring entropy is a simple and elegant way to characterize order or disorder in systems. We can use this measure as a way of characterizing the magnitude and nature of order in organization structures. Especially where electronic means of communication make it fairly easy to measure existence and density of communication between various players, it is also a measure that can rather easily be implemented.

Organizational entropy can be defined as:

$\varepsilon = - \Sigma\, [P_i \;*\; \log P_i]$ $\qquad (i = 1 \rightarrow m)$

where P_n is the probability that a certain state will occur, in our case: the probability that a certain interaction link (above the threshold) will exist.

If we consider the four cases in the example from paragraph 2, as a maximum N*(N-1)/2 = 45 links can exist (if we take every link as a two-way interaction). Using the formula we can now calculate the organizational entropy of the various examples:

- In the first example (unconnected nodes):

  $\varepsilon = -10 * (0 * \log 0) = 0$

- In the second example (hierarchical structure), 9 links exist:

  $\varepsilon = -10 * (9/45 * \log 9/45) = 1.40$

- In the third example (fully connected), 45 links exist:

  $\varepsilon = -10 * (45/45 * \log 45/45) = 0$

- In the fourth example (networked structure), 20 links exist:

  $\varepsilon = -10 * (20/45 * \log 20/45) = 1{,}57$

The situation in which no links exist and the situation in which all links exist span the extremes, and have no practical meaning in organizational terms. Of the other two examples, the hierarchical structure has the lowest organizational entropy, and hence represents a higher level of order than the networked structure example from figure 3.

We can see that networked structures as shown above, in terms of organizational entropy, are neatly positioned between structured order and total chaos. Hence networked organizations require a connectivity that is substantially higher than the procedural hierarchical organization, without ending into the other extreme where everything is connected to everything.

4. NETWORK MORPHOLOGY AND THE ENTROPY MEASURE

We have seen in paragraph 2 that connectivity and concentration determine network morphology. In paragraph 3 we showed that networks might be characterized by their morphology and their order / disorder. In this paragraph we connect the network morphology with network order.

In order to establish this relation between connectivity (characterized by a connectivity index Icn), concentration (characterized by the concentration index Icc) and network entropy ε, let us consider a network with n nodes (n>1), of which N nodes are fully connected to the other nodes (N<n) with a total number of connections in the network K (it follows that $K \geq N*(n-1)$).

We define:

- The concentration index Icc as N/n,
- The connectivity index Icn as K/n^2,
- Entropy ε as $- \Sigma\, [P_{ij} \;*\; \log P_{ij}]$, with $i = 1 \rightarrow m$, and $j = 1 \rightarrow m$, in which P_{ij} is the chance of the existence of connection $i \rightarrow j$.

Hence, entropy ε is $- \Sigma\, [P_{ij} \;*\; \log P_{ij}]$ for the fully connected nodes plus $- \Sigma\, [P_{ij} \;*\; \log P_{ij}]$ for the not fully connected nodes, or $\varepsilon = \varepsilon_N + \varepsilon_n$.

For the N fully connected nodes it goes:

- P_{ij} for the fully connected nodes is 1 (because for these nodes all connections exist),
- Therefore $\varepsilon_N = - N * [1 * \log 1] = 0$ (or, the contribution of the fully connected nodes to entropy is 0),

  For the n-N not fully connected nodes it goes:

- The number of remaining connections in the network is K – [N*(n-1)],
- Since the total possible number of connections in the network is n*(n-1), and since N*(n-1) connections are used up by the fully connected nodes, the total possible number of remaining connections is (n*(n-1) - N*(n-1)) or (n-N)*(n-1),
- P_{ij} for the not fully connected nodes is $[(K-N*(n-1))/((n-N)*(n-N-1))]$,
- Therefore $\varepsilon_n = - (n-N) * \{[(K-N*(n-1))/((n-N)*(n-N-1))] * \log [(K-N*(n-1))/((n-N)*(n-N-1))]\}$.

It follows that:

- The total network entropy $\varepsilon = \varepsilon_N + \varepsilon_n = 0 - (n-N)* \{[(K-N*(n-1))/((n-N)*(n-N-1))]*\log [(K-N*(n-1))/((n-N)*(n-N-1))]\}$.

Let us consider an example of a network of 1000 nodes (n = 1000) and a total number of 500.000 connections (K = 500.000). The connectivity for this network Icn = $500.000/ (1000)^2$ = 0,5. For various levels of the concentration index Icc the entropy is displayed graphically in figure 4 (note that in this case the maximum value for Icc is 0,5 as at this level all network connections are used up by the fully connected nodes (N)).

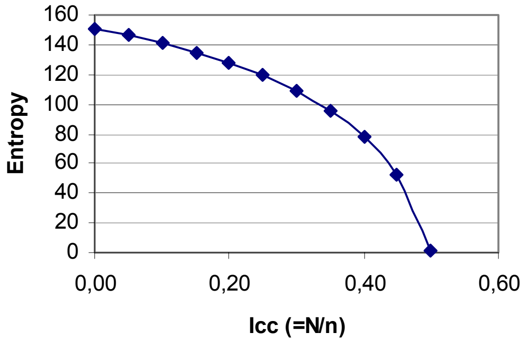

Fig. 4. Entropy versus concentration

Analogously, we can explore the relation between entropy and connectivity. In this case we fix Icc = N/n = 0,3. Hence N = 0,3 * 1000 = 300. For various Icn this yields values of K based on the formula Icn = K/n^2. This is displayed graphically in figure 5.

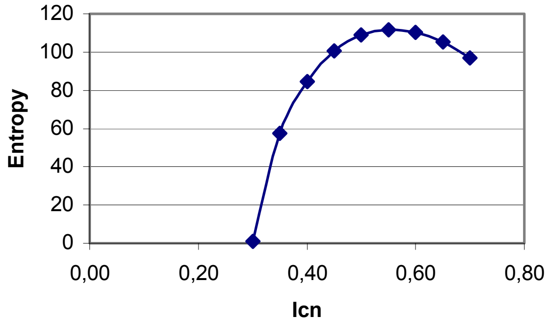

Fig. 5. Entropy versus connectivity

If the two parameters concentration (Icc) and connectivity (Icn) are combined, this yields figure 6. Note that in the bottom left area of figure 6 $P_{ij} < 0$, which is of course impossible. This is the area in which K is to small to cover all the connections necessary for N, let alone to leave free connections between the other n-N nodes.

Figure 6 demonstrates the strong increase of entropy if the relation between connectivity and concentration in a networked structure gets lost during transformation from a hierarchical structure to a networked structure. This phenomenon can be easily observed if, in a meeting between people with no historical relation, the chairman is suddenly removed. It takes quite a lot of time before some form of order is restored and one or a small number of people take the (informal) lead.

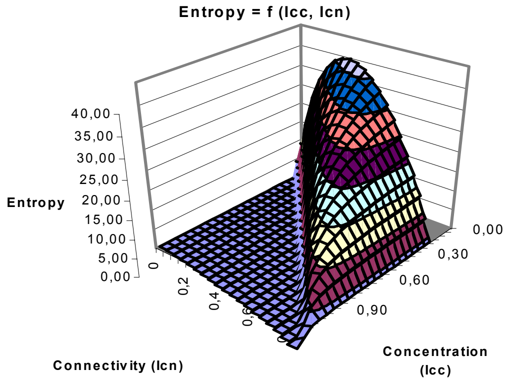

Fig. 6. Entropy as a function of concentration and connectivity

5. MORPHOLOGY, ENTROPY AND STABILITY

So far we have been concerned with a networked structure as a static configuration of nodes and links. However, if the networked structure is to be considered as a dynamic structure, reflecting some purpose to be fulfilled in relation to its relevant environment the morphology of connections between the nodes should be considered as a reflection of this purpose. This implies that they change over time in response to changing external requirements and the internal conditions to satisfy those. In this process positive feedback as well as negative feedback mechanisms are to be expected. The strength and balance between the two dependent on the actual level of connectivity and concentration in the network. Bureaucratic (high concentration/low connectivity) systems mainly yield negative feedback (steering the network to its intended behavior), fully connected networks (high connectivity, low concentration) will yield largely positive feedback as the change energy freely flows through the system.

Probably the simplest system that incorporates positive and negative feedback together a duplication function is May's [2] 'rabbit' formula (see figure 7), one that is widely quoted:

$X_{t+1} = X_t * \lambda * (1-X_t)$ 0<X<1

In this formula X changes dependent on the reproduction parameter λ, and counteracted by some force depicted as 1-X. It is the illustration of the evolution of an imaginary rabbit population May tried to predict the final size of this population, without limitations for size of population or time.

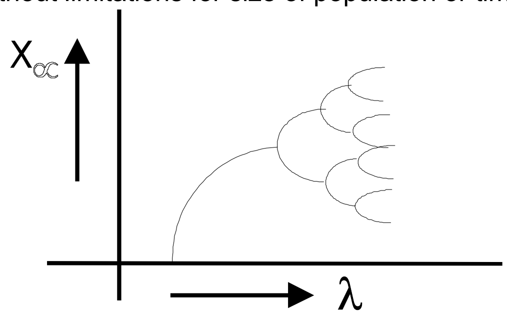

Fig. 7. May's rabbit population

This approach conceptualizes the network as a dynamic system, with the λ parameter as determinant of network complexity. The interesting question is whether these two concepts can be combined. In other words, is there a relation between the entropy parameter (with the under-lying connectivity and concentration parameters) and the reproduction parameter as in May's [2] formula?

In order to explore this let us imagine a networked system with say 20 nodes. And let us assume that those nodes are all linked to each other one by one, so the connections form a circle connecting all nodes. Now let's us assume that on t=1 one of the nodes becomes 'infected' by a condition that can be transmitted over the connections. At t=2, the adjacent nodes to the original ones will be infected. At t=3 the two sub-sequent nodes will be infected, and so on until all nodes are infected. The sequence of the number of infected nodes will look like:

1 – 3 – 5 – 7 – 9 –11 –13 –15- 17 – 19 – (20)

Alternatively, let us assume that all nodes are connected to each other through a central node (star configuration) and lets assume that on t=1 one becomes infected. If this infected node is the central one, at t=2 the whole population is infected. If the infected node is not the central node, at t=2 the central node will be infected, and at t=3 all remaining nodes will be infected.

The propagation of infections (e.g., the reproduction factor) of the network is heavily dependent on the morphology of the network. In the above example the connectivity-index is the same in both networks, whereas the concentration index is very high in the latter network while very low in the circular configuration.

It is easy to see that dependent on concentration and connectivity of the network (and hence the entropy measure) the λ parameter will vary as well. Yet deriving a general mathematical expression to express λ in terms of Icn and Icc, or, even better in entropy ε, proves to be a lot harder. Here it is sufficient to demonstrate the existence of the connection between the morphology parameters and the reproduction parameter.

6. CONCLUSIONS, IMPLICATIONS, AND FURTHER RESEARCH

From the above we may conclude that:

- It is possible to mathematically connect entropy (and hence network order / disorder) with the traditional network measures of connectivity and concentration (morphology),
- It seems in principle possible to connect network complexity (in terms of the λ reproduction parameter in the May formula) to network order/disorder (entropy).

There are some important implications to be derived from the above. The first is that management of organizational connectivity and concentration is crucial in keeping the network within a bandwidth between inflexible structured order and total chaos. The second is that if we assume the link between dynamic complexity (λ) and static entropy to be general, then there must be a direct connection between the connectivity and concentration of economic systems as e.g. stock markets and money markets, and the (in)stability which in practice can be observed in the behavior of such systems. Then, the sudden disruptions in exchange rates and stock prices might not (only) relate to changes in the underlying value creation mechanisms, but can (also) be considered as the 'spontaneous' property of the system itself. And hence, governance measures are conceivable at the level of the system, its connectivity and concentration, rather than interventions in the nodes (actors) themselves.

Further research should focus on establishing the formal mathematical connection between connectivity, concentration and entropy on the one hand and complexity (e.g., λ) on the other. While for a number of idealized cases (like pure hierarchical, pure circular and for a random distribution of connections over nodes) this is easily done, the current research challenge is to resolve the general case.

Literature

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